Proofs And Refutations The Logic Of Mathematical Discovery Imre Lakatos

Lakatos: An Introduction provides a thorough overview of both Lakatos's thought and his place in twentieth century philosophy. It is an essential and insightful read for students and anyone interested in the philosophy of science.

This book offers an excursion through the developmental area of research mathematics. It presents some 40 papers, published between the 1870s and the 1970s, on proofs of the Cantor-Bernstein theorem and the related Bernstein division theorem. While the emphasis is placed on providing accurate proofs, similar to the originals, the discussion is broadened to include aspects that pertain to the methodology of the development of mathematics and to the philosophy of mathematics. Works of prominent mathematicians and logicians are reviewed, including Cantor, Dedekind, Schröder, Bernstein, Borel, Zermelo, Poincaré, Russell, Peano, the Königs, Hausdorff, Sierpinski, Tarski, Banach, Brouwer and several others mainly of the Polish and the Dutch schools. In its attempt to present a diachronic narrative of one mathematical topic, the book resembles Lakatos' celebrated book Proofs and Refutations. Indeed, some of the observations made by Lakatos are corroborated herein. The analogy between the two books is clearly anything but superficial, as the present book also offers new theoretical insights into the methodology of the development of mathematics (proof-processing), with implications for the historiography of mathematics.

Proofs and RefutationsThe Logic of Mathematical DiscoveryCambridge University Press

Looks at the competition between French and Russian mathematicians over the nature of infinity during the twentieth century.

In recent years, many students have been introduced to topology in high school mathematics. Having met the Mobius band, the seven bridges of Konigsberg, Euler's polyhedron formula, and knots, the student is led to expect that these picturesque ideas will come to full flower in university topology courses. What a disappointment "undergraduate topology" proves to be! In most institutions it is either a service course for analysts, on abstract spaces, or else an introduction to homological algebra in which the only geometric activity is the completion of commutative diagrams. Pictures are kept to a minimum, and at the end the student still does nr~ understand the simplest topological facts, such as the rcason why knots exist. In my opinion, a well-balanced introduction to topology should stress its intuitive geometric aspect, while admitting the legitimate interest that analysts and algebraists have in the subject. At any rate, this is the aim of the present book. In support of this view, I have followed the historical development where practicable, since it clearly shows the influence of geometric thought at all stages. This is not to claim that topology received its main impetus from geometric recreations like the seven bridges; rather, it resulted from the l'isualization of problems from other parts of mathematics-complex analysis (Riemann), mechanics (Poincare), and group theory (Dehn). It is these connec tions to other parts of mathematics which make topology an important as well as a beautiful subject.

This inaugural handbook documents the distinctive research field that utilizes history and philosophy in investigation of theoretical, curricular and pedagogical issues in the teaching of science and mathematics. It is contributed to by 130 researchers from 30 countries; it provides a logically structured, fully referenced guide to the ways in which science and mathematics education is, informed by the history and philosophy of these disciplines, as well as by the philosophy of education more generally. The first handbook to cover the field, it lays down a much-needed marker of progress to date and provides a platform for informed and coherent future analysis and research of the subject. The publication comes at a time of heightened worldwide concern over the standard of science and mathematics education, attended by fierce debate over how best to reform curricula and enliven student engagement in the subjects. There is a growing recognition among educators and policy makers that the learning of science must dovetail with learning about science; this handbook is uniquely positioned as a locus for the discussion. The handbook features sections on pedagogical, theoretical, national, and biographical research, setting the literature of each tradition in its historical context. It reminds readers at a crucial juncture that there has been a long and rich tradition of historical and philosophical engagements with science and mathematics teaching, and that lessons can be learnt from these engagements for the resolution of current theoretical, curricular and pedagogical questions that face teachers and administrators. Science educators will be grateful for this unique, encyclopaedic handbook, Gerald Holton, Physics Department, Harvard University This handbook gathers the fruits of over thirty years' research by a growing international and cosmopolitan community Fabio Bevilacqua, Physics Department, University of Pavia

Proof Theory of Modal Logic is devoted to a thorough study of proof systems for modal logics, that is, logics of necessity, possibility, knowledge, belief, time, computations etc. It contains many new technical results and presentations of novel proof procedures. The volume is of immense importance for the interdisciplinary fields of logic, knowledge representation, and automated deduction.

This volume contains articles covering a broad spectrum of proof theory, with an emphasis on its mathematical aspects. The articles should not only be interesting to specialists of proof theory, but should also be accessible to a diverse audience, including logicians, mathematicians, computer scientists and philosophers. Many of the central topics of proof theory have been included in a self-contained expository of articles, covered in great detail and depth. The chapters are arranged so that the two introductory articles come first; these are then followed by articles from core classical areas of proof theory; the handbook concludes with articles that deal with topics closely related to computer science.

This collection exhibits and confirms the originality, range and the essential unity of his work.

Imre Lakatos (1922-1974) was one of the protagonists in shaping the "new philosophy of science". More than 25 years after his untimely death, it is time for a critical re-evaluation of his ideas. His main theme of locating rationality within the scientific process appears even more compelling today, after many historical case studies have revealed the cultural and societal elements within scientific practices. Recently there has been, above all, an increasing interest in Lakatos' philosophy of mathematics, which emphasises heuristics and mathematical practice over logical justification. But suitable modifications of his approach are called for in order to make it applicable to modern axiomatised theories. Pioneering historical research in England and Hungary has unearthed hitherto unknown facts about Lakatos' personal life, his wartime activities and his involvement in the political developments of post-war Europe. From a communist activist committed to Györgyi Lukács' thinking, Lakatos developed into a staunch anti-Marxist who found his intellectual background in Popper's critical rationalism. The volume also publishes for the first time a part of his Debrecen Ph.D. thesis and it is concluded by a bibliography of his Hungarian writings.

This short book, geared towards undergraduate students of computer science and mathematics, is specifically designed for a first course in mathematical logic. A proof of Gödel's completeness theorem and its main consequences is given using Robinson's completeness theorem and Gödel's compactness theorem for propositional logic. The reader will familiarize himself with many basic ideas and artifacts of mathematical logic: a non-ambiguous syntax, logical equivalence and consequence relation, the Davis-Putnam procedure, Tarski semantics, Herbrand models, the axioms of identity, Skolem normal forms, nonstandard models and, interestingly enough, proofs and refutations viewed as graphic objects. The mathematical prerequisites are minimal: the book is accessible to anybody having some familiarity with proofs by induction. Many exercises on the relationship between natural language and formal proofs make the book also interesting to a wide range of students of philosophy and linguistics.

Proofs and Refutations is essential reading for all those interested in the methodology, the philosophy and the history of mathematics. Much of the book takes the form of a discussion between a teacher and his students. They propose various solutions to some mathematical problems and investigate the strengths and weaknesses of these solutions. Their discussion (which mirrors certain real developments in the history of mathematics) raises some philosophical problems and some problems about the nature of mathematical discovery or creativity. Imre Lakatos is concerned throughout to combat the classical picture of mathematical development as a steady accumulation of established truths. He shows that mathematics grows instead through a richer, more dramatic process of the successive improvement of creative hypotheses by attempts to 'prove' them and by criticism of these attempts: the logic of proofs and refutations.

The twentieth century has witnessed an unprecedented 'crisis in the foundations of mathematics', featuring a world-famous paradox (Russell's Paradox), a challenge to 'classical' mathematics from a world-famous mathematician (the 'mathematical intuitionism' of Brouwer), a new foundational school (Hilbert's Formalism), and the profound incompleteness results of Kurt Gödel. In the same period, the cross-fertilization of mathematics and philosophy resulted in a new sort of 'mathematical philosophy', associated most notably (but in different ways) with Bertrand Russell, W. V. Quine, and Gödel himself, and which remains at the focus of Anglo-Saxon philosophical discussion. The present collection brings together in a convenient form the seminal articles in the philosophy of mathematics by these and other major thinkers. It is a substantially revised version of the edition first published in 1964 and includes a revised bibliography. The volume will be welcomed as a major work of reference at this level in the field.

This text develops a comprehensive theory of programming languages based on type systems and structural operational semantics. Language concepts are precisely defined by their static and dynamic semantics, presenting the essential tools both intuitively and rigorously while relying on only elementary mathematics. These tools are used to analyze and prove properties of languages and provide the framework for combining and comparing language features. The broad range of concepts includes fundamental data types such as sums and products, polymorphic and abstract types, dynamic typing, dynamic dispatch, subtyping and refinement types, symbols and dynamic classification, parallelism and cost semantics, and concurrency and distribution. The methods are directly applicable to language implementation, to the development of logics for reasoning about programs, and to the formal verification language properties such as type safety. This thoroughly revised second edition includes exercises at the end of nearly every chapter and a new chapter on type refinements.

Volume I brings together his very influential but scattered papers on the philosophy of the physical sciences, and includes one important unpublished essay on the effect of Newton's scientific achievement. Volume 2 presents his work on the philosophy of mathematics together with some critical essays on contemporary philosophers of science.

2014 Reprint of 1954 American Edition. Full facsimile of the original edition, not reproduced with Optical Recognition Software. This two volume classic comprises two titles: "Patterns of Plausible Inference" and "Induction and Analogy in Mathematics." This is a guide to the practical art of plausible reasoning, particularly in mathematics, but also in every field of human activity. Using mathematics as the example par excellence, Polya shows how even the most rigorous deductive discipline is heavily dependent on techniques of guessing, inductive reasoning, and reasoning by analogy. In solving a problem, the answer must be guessed at before a proof can be given, and guesses are usually made from a knowledge of facts, experience, and hunches. The truly creative mathematician must be a good guesser first and a good prover afterward; many important theorems have been guessed but no proved until much later. In the same way, solutions to problems can be guessed, and a god guesser is much more likely to find a correct solution. This work might have been called "How to Become a Good Guesser."-From the Dust Jacket.

Extends the ideas of social constructivism to the philosophy of mathematics, developing a powerful critique of traditional absolutist conceptions of mathematics, and proposing a reconceptualization of the philosophy of mathematics.

Proofs and Refutations is for those interested in the methodology, philosophy and history of mathematics.

Five Proofs of the Existence of GodÊprovides a detailed, updated exposition and defense of five of the historically most important (but in recent years largely neglected) philosophical proofs of God's existence: the Aristotelian proof, the Neo-Platonic proof, the Augustinian proof, the Thomistic proof, and the Rationalist proof. Ê This book also offers a detailed treatment of each of the key divine attributes -- unity, simplicity, eternity, omnipotence, omniscience, perfect goodness, and so forth -- showing that they must be possessed by the God whose existence is demonstrated by the proofs.Ê Finally, it answers at length all of the objections that have been leveled against these proofs. Ê This book offers as ambitious and complete a defense of traditional natural theology as is currently in print.Ê Its aim is to vindicate the view of the greatest philosophers of the past -- thinkers like Aristotle, Plotinus, Augustine, Aquinas, Leibniz, and many others -- that the existence of God can be established with certainty by way of purely rational arguments.Ê It thereby serves as a refutation both of atheism and of the fideism which gives aid and comfort to atheism. Ê

This book presents chapters exploring the most recent developments in the role of technology in proving. The full range of topics related to this theme are explored, including computer proving, digital collaboration among mathematicians, mathematics teaching in schools and universities, and the use of the internet as a site of proof learning. Proving is sometimes thought to be the aspect of mathematical activity most resistant to the influence of technological change. While computational methods are well known to have a huge importance in applied mathematics, there is a perception that mathematicians seeking to derive new mathematical results are unaffected by the digital era. The reality is quite different. Digital technologies have transformed how mathematicians work together, how proof is taught in schools and universities, and even the nature of proof itself. Checking billions of cases in extremely large but finite sets, impossible a few decades ago, has now become a standard method of proof. Distributed proving, by teams of mathematicians working independently on sections of a problem, has become very much easier as digital communication facilitates the sharing and comparison of results. Proof assistants and dynamic proof environments have influenced the verification or refutation of conjectures, and ultimately how and why proof is taught in schools. And techniques from computer science for checking the validity of programs are being used to verify mathematical proofs. Chapters in this book include not only research reports and case studies, but also theoretical essays, reviews of the state of the art in selected areas, and historical studies. The authors are experts in the field.

In the four decades since Imre Lakatos declared mathematics a "quasi-empirical science," increasing attention has been paid to the process of proof and argumentation in the field -- a development paralleled by the rise of computer technology and the mounting interest in the logical underpinnings of mathematics. Explanantion and Proof in Mathematics assembles perspectives from mathematics education and from the philosophy and history of mathematics to strengthen mutual awareness and share recent findings and advances in their interrelated fields. With examples ranging from the geometrists of the 17th century and ancient Chinese algorithms to cognitive psychology and current educational practice, contributors explore the role of refutation in generating proofs, the varied links between experiment and deduction, the use of diagrammatic thinking in addition to pure logic, and the uses of proof in mathematics education (including a critique of "authoritative" versus "authoritarian" teaching styles). A sampling of the coverage: The conjoint origins of proof and theoretical physics in ancient Greece. Proof as bearers of mathematical knowledge. Bridging knowing and proving in mathematical reasoning. The role of mathematics in long-term cognitive development of reasoning. Proof as experiment in the work of Wittgenstein. Relationships between mathematical proof, problem-solving, and explanation. Explanation and Proof in Mathematics is certain to attract a wide range of readers, including mathematicians, mathematics education professionals, researchers, students, and philosophers and historians of mathematics.

This book-length treatment provides a unified account of what is distinctive in the ancient approach to the self-refutation argument.

Logic concepts are more mainstream than you may realize. There's logic every place you look and in almost everything you do, from deciding which shirt to buy to asking your boss for a raise, and even to watching television, where themes of such shows as CSI and Numbers incorporate a variety of logistical studies. Logic For Dummies explains a vast array of logical concepts and processes in easy-to-understand language that make everything clear to you, whether you're a college student of a student of life. You'll find out about: Formal Logic Syllogisms Constructing proofs and refutations Propositional and predicate logic Modal and fuzzy logic Symbolic logic Deductive and inductive reasoning Logic For Dummies tracks an introductory logic course at the college level. Concrete, real-world examples help you understand each concept you encounter, while fully worked out proofs and fun logic problems encourage you students to apply what you've learned.

In mathematics, a proof is a deductive argument for a mathematical statement. In the argument, other previously established statements, such as theorems, can be used. In principle, a proof can be traced back to self-evident or assumed statements, known as axioms. Proofs are examples of deductive reasoning and are distinguished from inductive or empirical arguments; a proof must demonstrate that a statement is always true (occasionally by listing all possible cases and showing that it holds in each), rather than enumerate many confirmatory cases. An unproved proposition that is believed true is known as a conjecture. Proofs employ logic but usually include some amount of natural language which usually admits some ambiguity. In fact, the vast majority of proofs in written mathematics can be considered as applications of rigorous informal logic. Purely formal proofs, written in symbolic language instead of natural language, are considered in proof theory. This book contains 'solutions' to some of the most noteworthy mathematical proofs (QED).

The traditional debate among philosophers of mathematics is whether there is an external mathematical reality, something out there to be discovered, or whether mathematics is the product of the human mind. This provocative book, now available in a revised and expanded paperback edition, goes beyond foundationalist questions to offer what has been called a "postmodern" assessment of the philosophy of mathematics--one that addresses issues of theoretical importance in terms of mathematical experience. By bringing together essays of leading philosophers, mathematicians, logicians, and computer scientists, Thomas Tymoczko reveals an evolving effort to account for the nature of mathematics in relation to other human activities. These accounts include such topics as the history of mathematics as a field of study, predictions about how computers will influence the future organization of mathematics, and what processes a proof undergoes before it reaches publishable form. This expanded edition now contains essays by Penelope Maddy, Michael D. Resnik, and William P. Thurston that address the nature of mathematical proofs. The editor has provided a new afterword and a supplemental bibliography of recent work.

The work that helped to determine Paul Feyerabend's fame and notoriety, Against Method, stemmed from Imre Lakatos's challenge: "In 1970 Imre cornered me at a party. 'Paul,' he said, 'you have such strange ideas. Why don't you write them down? I shall write a reply, we publish the whole thing and I promise you—we shall have a lot of fun.' " Although Lakatos died before he could write his reply, For and Against Method reconstructs his original counter-arguments from lectures and correspondence previously unpublished in English, allowing us to enjoy the "fun" two of this century's most eminent philosophers had, matching their wits and ideas on the subject of the scientific method. For and Against Method opens with an imaginary dialogue between Lakatos and Feyerabend, which Matteo Motterlini has constructed, based on their published works, to synthesize their positions and arguments. Part one presents the transcripts of the last lectures on method that Lakatos delivered. Part two, Feyerabend's response, consists of a previously published essay on anarchism, which began the attack on Lakatos's position that Feyerabend later continued in Against Method. The third and longest section consists of the correspondence Lakatos and Feyerabend exchanged on method and many other issues and ideas, as well as the events of their daily lives, between 1968 and Lakatos's death in 1974. The delight Lakatos and Feyerabend took in philosophical debate, and the relish with which they sparred, come to life again in For and Against Method, making it essential and lively reading for anyone interested in these two fascinating and controversial thinkers and their immense contributions to philosophy of science. "The writings in this volume are of considerable intellectual importance, and will be of great interest to anyone concerned with the development of the philosophical views of Lakatos and Feyerabend, or indeed with the development of philosophy of science in general during this crucial period."—Donald Gillies, British Journal for the Philosophy of Science (on the Italian edition) "A stimulating exchange of letters between two philosophical entertainers."—Tariq Ali, The Independent Imre Lakatos (1922-1974) was professor of logic at the London School of Economics. He was the author of Proofs and Refutations and the two-volume Philosophical Papers. Paul Feyerabend (1924-1994) was educated in Europe and held numerous teaching posts throughout his career. Among his books are Against Method; Science in a Free Society; Farewell to Reason; and Killing Time: The Autobiography of Paul Feyerabend, the last published by the University of Chicago Press.

In case you are considering to adopt this book for courses with over 50 students, please contact ties.nijssen@springer.com for more information. This introduction to mathematical logic starts with propositional calculus and first-order logic. Topics covered include syntax, semantics, soundness, completeness, independence, normal forms, vertical paths through negation normal formulas, compactness, Smullyan's Unifying Principle, natural deduction, cut-elimination, semantic tableaux, Skolemization, Herbrand's Theorem, unification, duality, interpolation, and definability. The last three chapters of the book provide an introduction to type theory (higher-order logic). It is shown how various mathematical concepts can be formalized in this very expressive formal language. This expressive notation facilitates proofs of the classical incompleteness and undecidability theorems which are very elegant and easy to understand. The discussion of semantics makes clear the important distinction between standard and nonstandard models which is so important in understanding puzzling phenomena such as the incompleteness theorems and Skolem's Paradox about countable models of set theory. Some of the numerous exercises require giving formal proofs. A computer program called ETPS which is available from the web facilitates doing and checking such exercises. Audience: This volume will be of interest to mathematicians, computer scientists, and philosophers in universities, as well as to computer scientists in industry who wish to use higher-order logic for hardware and software specification and verification.

In this ambitious study, David Corfield attacks the widely held view that it is the nature of mathematical knowledge which has shaped the way in which mathematics is treated philosophically and claims that contingent factors have brought us to the present thematically limited discipline. Illustrating his discussion with a wealth of examples, he sets out a variety of approaches to new thinking about the philosophy of mathematics, ranging from an exploration of whether computers producing mathematical proofs or conjectures are doing real mathematics, to the use of analogy, the prospects for a Bayesian confirmation theory, the notion of a mathematical research programme and the ways in which new concepts are justified. His inspiring book challenges both philosophers and mathematicians to develop the broadest and richest philosophical resources for work in their disciplines and points clearly to the ways in which this can be done.

The ability to learn is one of the most fundamental attributes of intelligent behavior. Consequently, progress in the theory and computer modeling of learn ing processes is of great significance to fields concerned with understanding in telligence. Such fields include cognitive science, artificial intelligence, infor mation science, pattern recognition, psychology, education, epistemology, philosophy, and related disciplines. The recent observance of the silver anniversary of artificial intelligence has been heralded by a surge of interest in machine learning-both in building models of human learning and in understanding how machines might be endowed with the ability to learn. This renewed interest has spawned many new research projects and resulted in an increase in related scientific activities. In the summer of 1980, the First Machine Learning Workshop was held at Carnegie-Mellon University in Pittsburgh. In the same year, three consecutive issues of the Inter national Journal of Policy Analysis and Information Systems were specially devoted to machine learning (No. 2, 3 and 4, 1980). In the spring of 1981, a special issue of the SIGART Newsletter No. 76 reviewed current research projects in the field. . This book contains tutorial overviews and research papers representative of contemporary trends in the area of machine learning as viewed from an artificial intelligence perspective. As the first available text on this subject, it is intended to fulfill several needs.

Introduces an original approach to foundations of mathematics, departing from Gödel and Tarski and spanning many different areas of logic.

Proof and Disproof in Formal Logic is a lively and entertaining introduction to formal logic providing an excellent insight into how a simple logic works. This book concentrates on using logic as a tool: making and using formal proofs and disproofs of particular logical claims. The logic it uses - natural deduction - is very simple and shows how large mathematical universes can be built on small foundations. Aimed at undergraduates and graduates in computerscience, logic, mathematics, and philosophy, the text includes reference to...

This book argues against the view that mathematical knowledge is a priori, contending that mathematics is an empirical science and develops historically, just as natural sciences do. Kitcher presents a complete, systematic, and richly detailed account of the nature of mathematical knowledgeand its historical development, focusing on such neglected issues as how and why mathematical language changes, why certain questions assume overriding importance, and how standards of proof are modified.

Imre Lakatos's Proofs and Refutations is an enduring classic, which has never lost its relevance. Taking the form of a dialogue between a teacher and some students, the book considers various solutions to mathematical problems and, in the process, raises important questions about the nature of mathematical discovery and methodology. Lakatos shows that mathematics grows through a process of improvement by attempts at proofs and critiques of these attempts, and his work continues to inspire mathematicians and philosophers aspiring to develop a philosophy of mathematics that accounts for both the static and the dynamic complexity of mathematical practice. With a specially commissioned Preface written by Paolo Mancosu, this book has been revived for a new generation of readers.

This advanced text for undergraduate and graduate students introduces mathematical logic with an emphasis on proof theory and procedures for algorithmic construction of formal proofs. The self-contained treatment is also useful for computer scientists and mathematically inclined readers interested in the formalization of proofs and basics of automatic theorem proving. Topics include propositional logic and its resolution, first-order logic, Gentzen's cut elimination theorem and applications, and Gentzen's sharpened Hauptsatz and Herbrand's theorem. Additional subjects include resolution in first-order logic; SLD-resolution, logic programming, and the foundations of PROLOG; and many-sorted first-order logic. Numerous problems appear throughout the book, and two Appendixes provide practical background information.

Now available in a one-volume paperback, this book traces the development of the most important mathematical concepts, giving special attention to the lives and thoughts of such mathematical innovators as Pythagoras, Newton, Poincare, and Godel. Beginning with a Sumerian short story--ultimately linked to modern digital computers--the author clearly introduces concepts of binary operations; point-set topology; the nature of post-relativity geometries; optimization and decision processes; ergodic theorems; epsilon-delta arithmetization; integral equations; the beautiful "ideals" of Dedekind and Emmy Noether; and the importance of "purifying" mathematics. Organizing her material in a conceptual rather than a chronological manner, she integrates the traditional with the modern, enlivening her discussions with historical and biographical detail.

Described by the philosopher A.J. Ayer as a work of 'great originality and power', this book revolutionized contemporary thinking on science and knowledge. Ideas such as the now legendary doctrine of 'falsificationism' electrified the scientific community, influencing even working scientists, as well as post-war philosophy. This astonishing work ranks alongside The Open Society and Its Enemies as one of Popper's most enduring books and contains insights and arguments that demand to be read to this day.